

# Dynamic Modeling and Robust Fault Detection of Gas Turbine Engines

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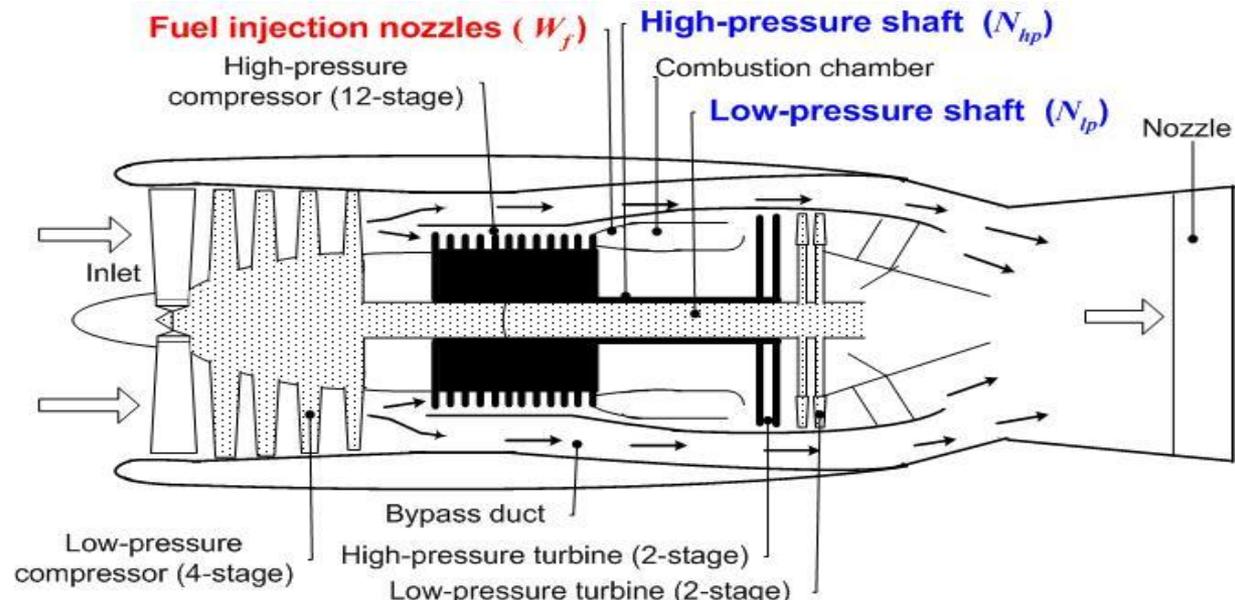
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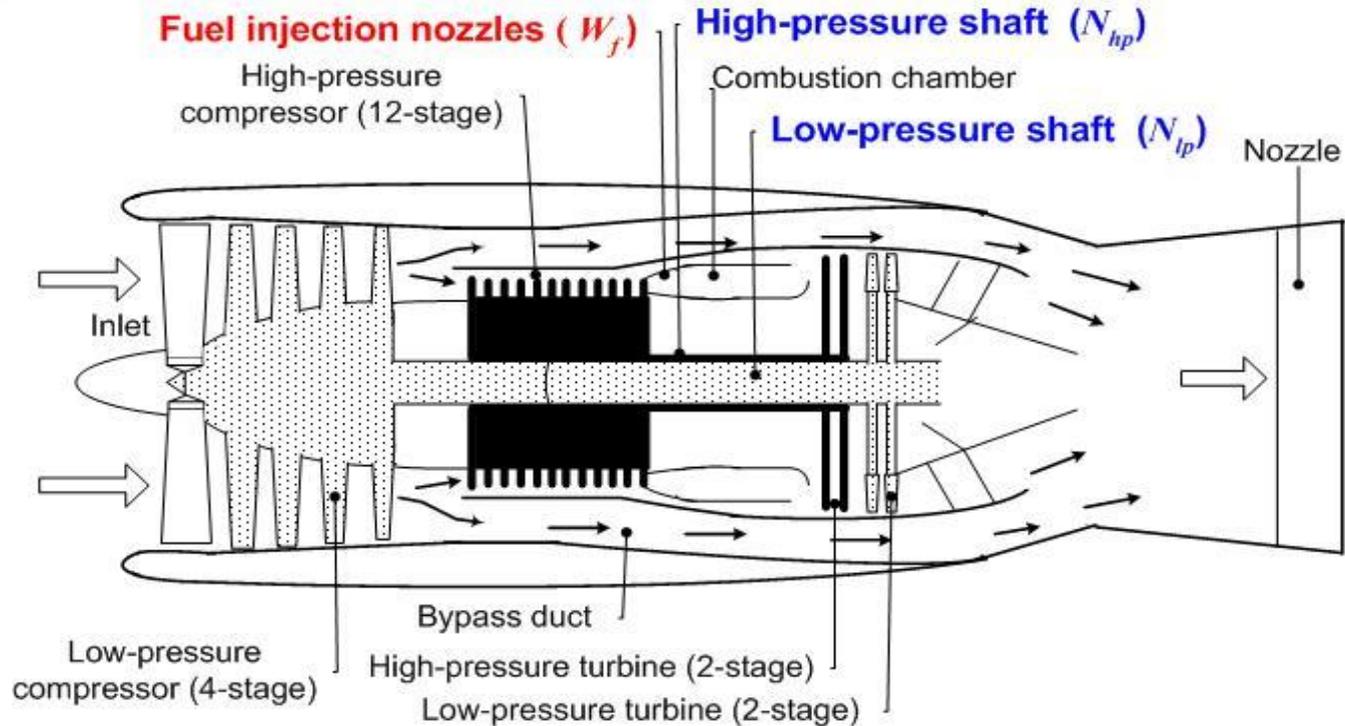
# Outline

- **1. Problem Statement and Objectives**
- **2. Output Error Modelling**
- **3. Robust Fault Detection Observer Design**
- **4. Application and Results**

# 1. Problem statement

- Difficulties in on-board modelling and condition monitoring
  1. An Output Error (OE) model is required
  2. Conflicts between iterative identification algorithms and limited on-board computation resources
  3. Being corrupted by the disturbance and/or noise.





Around some operating point

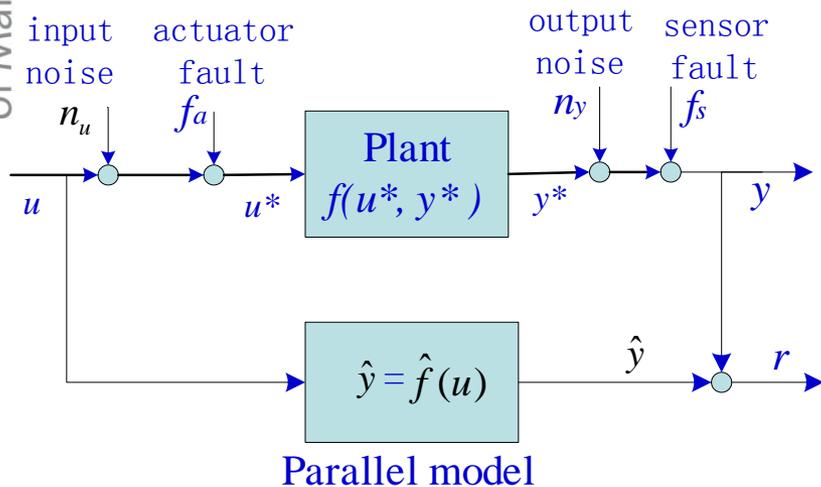
- The reheat nozzle area and the angle of the VGVs (Variable guide vanes) are fixed.
- The fuel flow ( $W_f$ ) is the input:  $u(t) = W_f(t)$
- The shaft speeds are the primary outputs:  
 $y(t) = [y_1, y_2] = [N_{LP}, N_{HP}]$

# Objectives

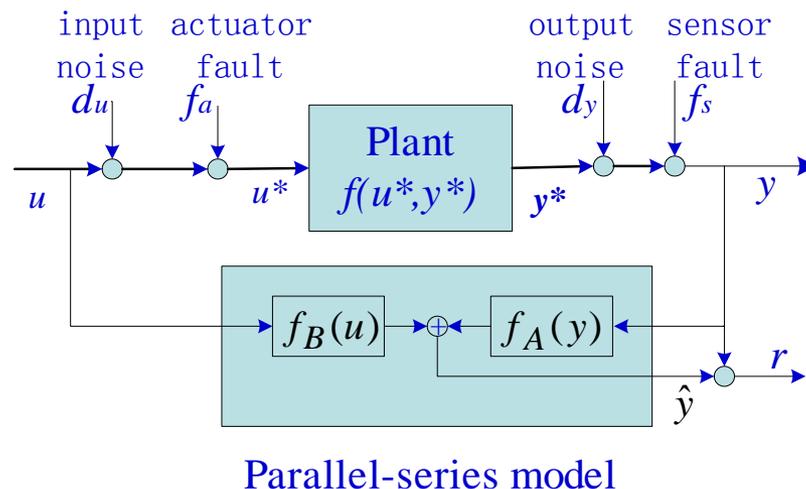
- Fitting the parametric model to the data:  
fast identification algorithm (accelerating the convergence speed)
- Designing the observer (residual generator):  
the residual should be sensitive to the fault, but robust to the disturbances.

# 2. EE v.s. OE

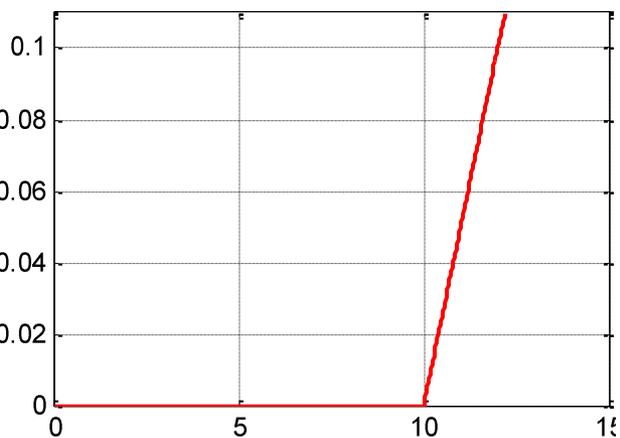
(a) parallel connection mode



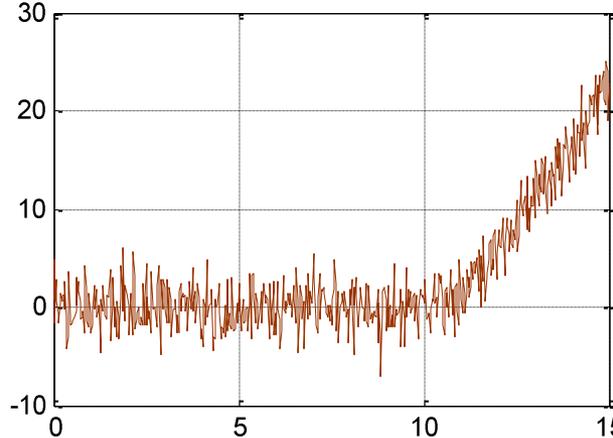
(b) parallel-series connection mode



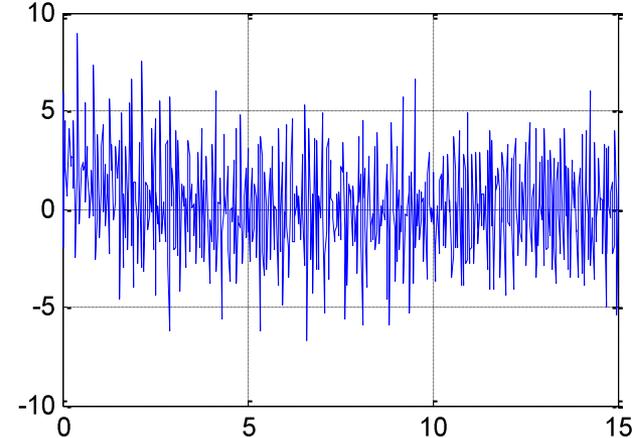
Ramp Fault



Ramp Fault



Ramp Fault



# OE Identification

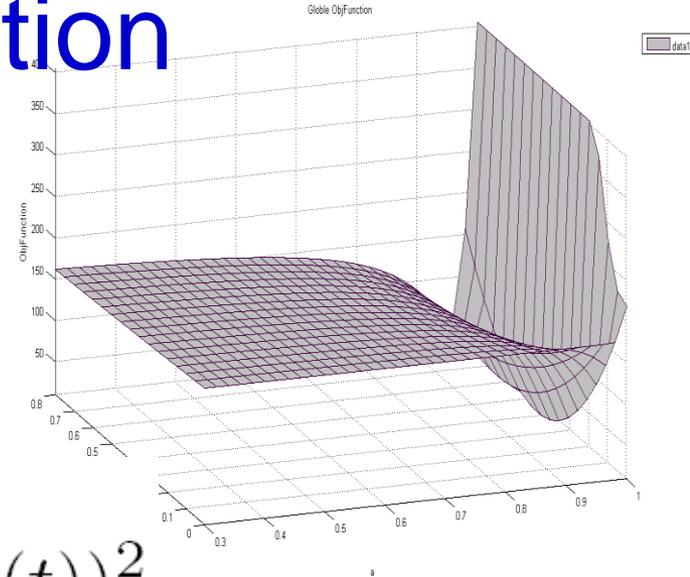
$$\hat{y}_i(t) = \frac{\sum_{j=1}^n \beta_{i,j} q^{-j}}{1 + \sum_{j=1}^n \alpha_{i,j} q^{-j}} u(t)$$

- Aims: minimize MSOE

$$\min_{\theta_i \in \mathbb{R}^{2n}} E(\theta_i) = \sum_{t=1}^N (y_i(t) - \theta_i^T \cdot \hat{z}_i(t))^2$$

- The objective function  $E(\theta_i)$  is highly nonlinear
- Some iterative optimization method has to be used, NLS (Nonlinear Least Squares)

$$\theta_i(k+1) = \theta_i(k) - \eta^* \cdot [\mathbf{R}_i(k)]^{-1} \cdot \frac{\partial E(\theta_i(k))}{\partial \theta_i}$$



# Correlated Output Errors

## a.) Iterative calculation of gradient

$$g_i(t) = \frac{\partial(y_i(t) - \hat{y}_i(t))}{\partial\theta_i}$$

$$= -\hat{z}_i(t) + [g_i(t-1) \cdots g_i(t-n) \ 0 \cdots 0] \cdot \theta_i(k)$$

and

$$\frac{\partial E(\theta_i(k))}{\partial\theta_i} = 2 \sum_{t=1}^N \varepsilon_i(t) \cdot g_i(t)$$

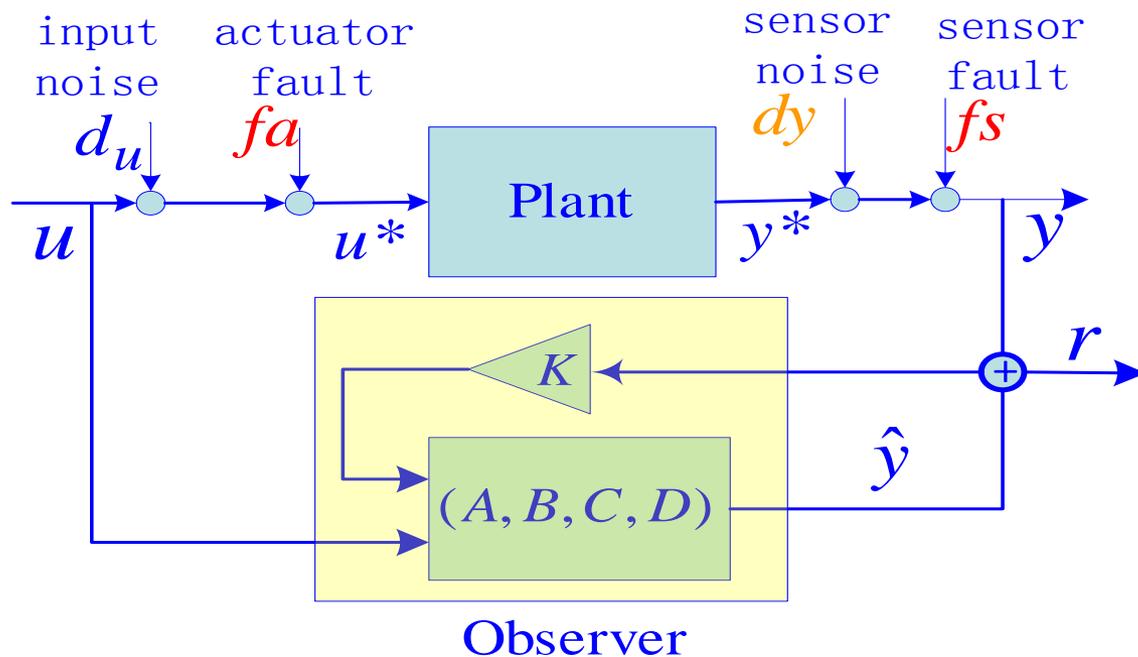
## b.) Approximation of $R_i(k)$

--Gauss-Newton method

$$\mathbf{R}_i(k) \approx \mathbf{H}_i(\theta_i(k)) \approx \mathbf{J}_i^T(k) \cdot \mathbf{J}_i(k)$$



### 3. Robust Fault Detection Observer Design



$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + B_d d(k) + B_f f(k) \\ y(k) = Cx(k) + D_d d(k) + D_f f(k) \end{cases}$$



$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + Bu(k) + K(y(k) - \hat{y}(k)) \\ \hat{y}(k) = C\hat{x}(k) + Du(k) \end{cases}$$

## TFMs (Transfer Function Matrices)

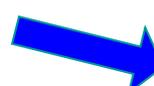
- The TFMs relating the residual  $r(z)$  to the fault  $f(z)$  and disturbance  $d(z)$  are

$$r(z) = G_f(z)f(z) + G_d(z)d(z)$$

$$\min J = \frac{J_1}{J_2}$$

Robustness index


 $\min \|G_d(z)\|_{z=e^{j\omega_r}}$


 $\max \| [G_f(z)] \|_{z=1}$

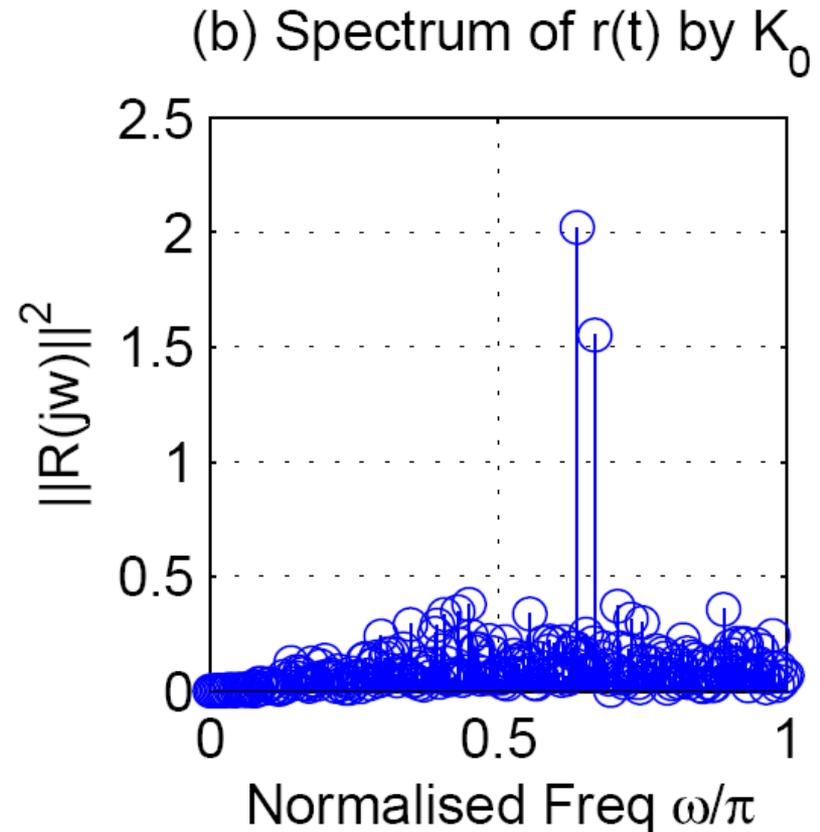
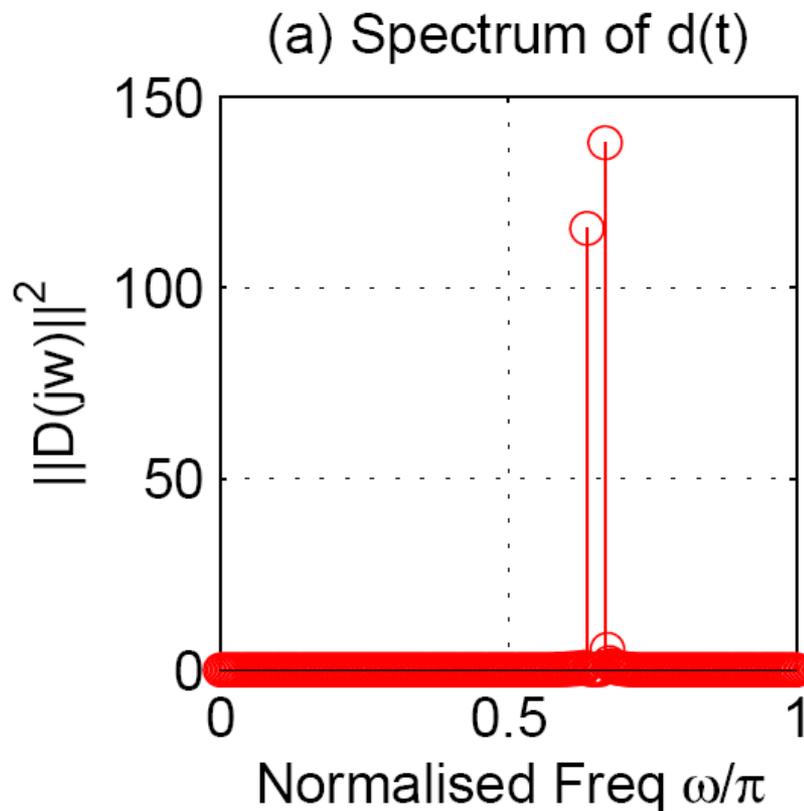
Sensitivity index

# Eigenstructure Assignment and Optimisation

- With the aid of eigen value analysis, the TFMs  $G_f(z)$ ,  $G_d(z)$  and the gain matrix  $\mathbf{K}$  can be parameterized with a set of eigenvalues and a set of free parameters.
- Using optimisation algorithms results in an optimal observer for fault detection and disturbance attenuation.

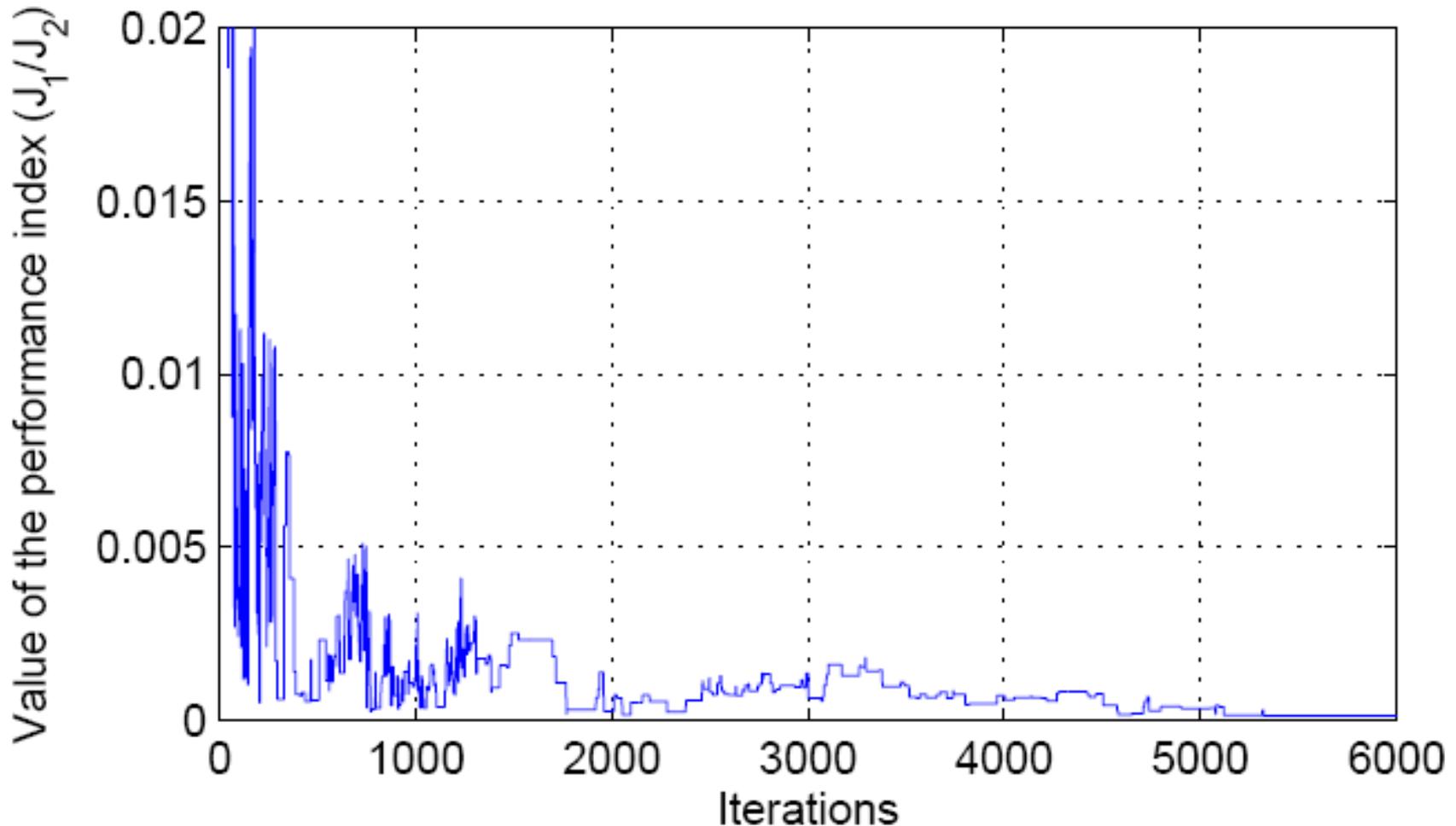
## 4. Results

- Disturbances injected into the system
- Residual spectrum analysis



# Trace of cost function over iteration -Simulated Annealing

The trace of cost function over iterations

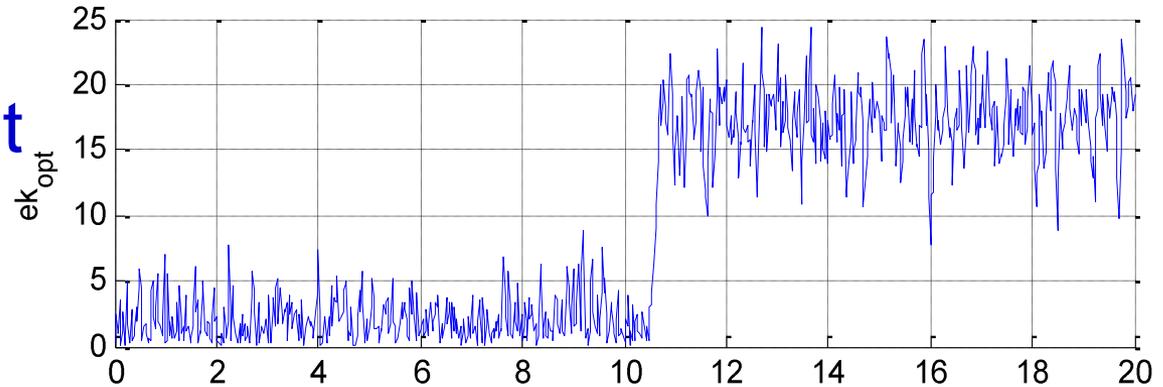


# Fault detection

residuals

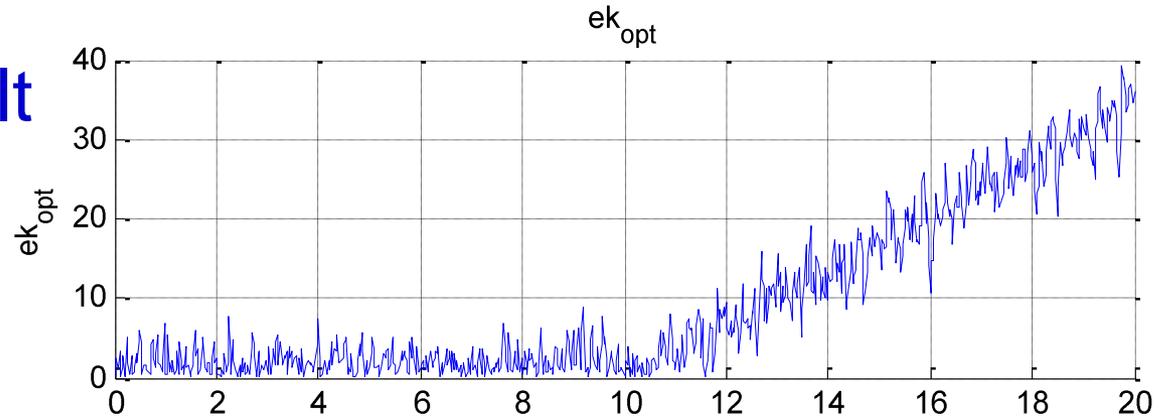
abrupt sensor fault

$$f_s(t) = \begin{cases} 0 & t < 10.5 \text{ sec} \\ 0.25 & t > 10.5 \text{ sec} \end{cases}$$

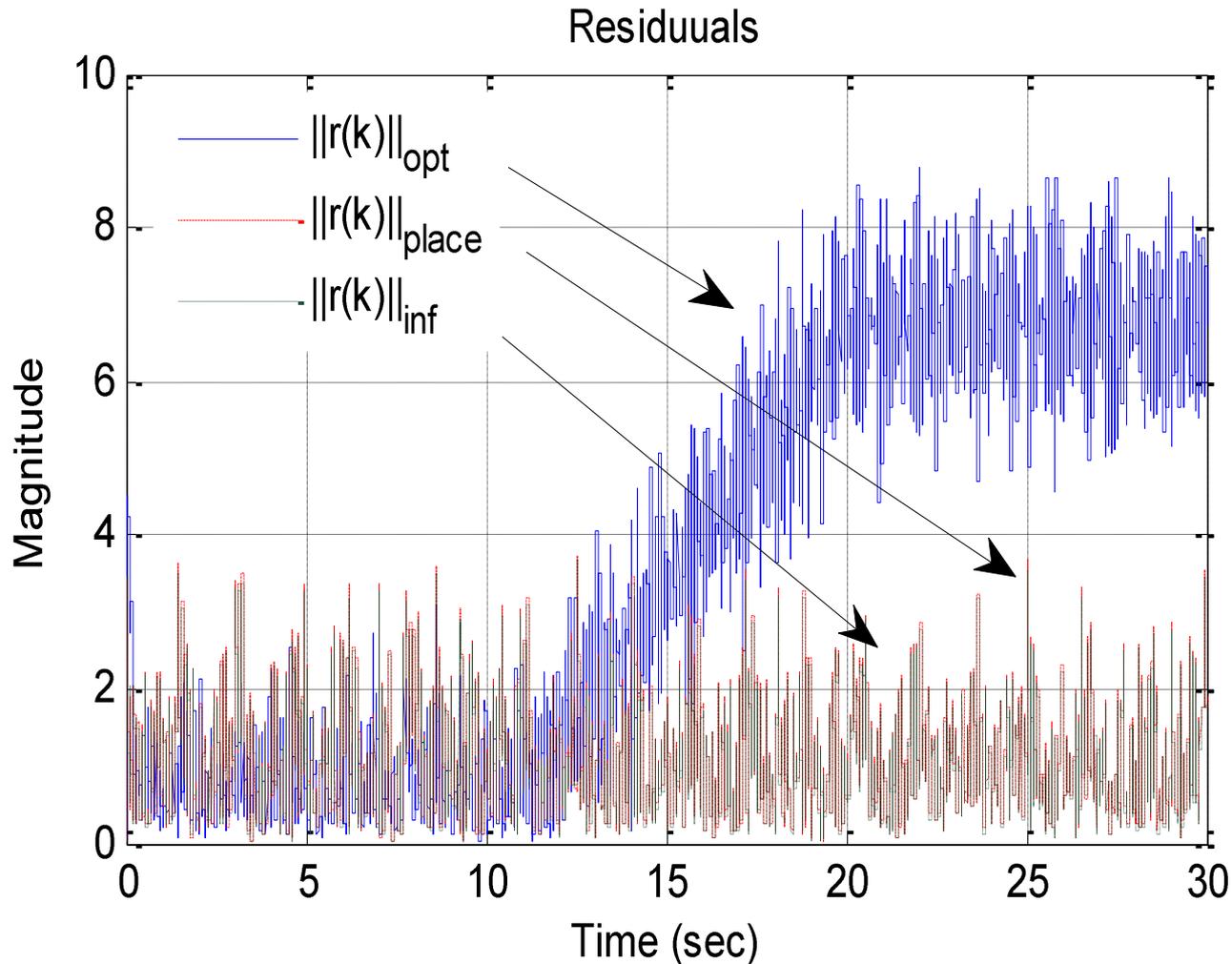


incipient sensor fault

$$f_s(t) = \begin{cases} 0 & t < 10.5 \text{ sec} \\ 0.5(t - 10.5) & t > 10.5 \text{ sec} \end{cases}$$



# Fault Detection



*Thanks*