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Dynamic Modelling and Robust Fault Detection of a Gas Turbine Engine

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Abstract— **Dynamic modelling and fault detection play an important role in the condition monitoring of gas turbine engines (GTEs). Although system identification and Robust Fault Detection Observer (RFDO) have been studied intensively, on-board fault detection raises challenges. A fast identification and discrete observer design is required because of the limited computation ability. In this paper, an output error model is identified first and a discrete observer is designed to avoid the discrete-continuous conversion. With the aid of disturbance frequency estimation, an improved performance index and a fast left-eigenvector based robust observer design method are proposed. As illustrated in the application results, a better disturbance attenuation and fault detection performance have been achieved.**

I. INTRODUCTION

With the fast development of computer techniques, modern aircraft gas turbine engines are controlled by full authority digital engine controllers (FADEC), which makes it possible to identify the dynamic model in real time and detect the engine fault on-board. Because of the limitation of on-board computation resources, fast dynamic modelling and fault detection design are required. Although they have been studied for several decades, system modelling and fault detection are treated separately in most of the literatures. The field of 'real-time output error modelling and fault detection of GTEs' is still relative young. In this paper, a combined fast identification and observer design method are proposed to bridge the gap.

On the GTE modelling, engine parameters may deviate from the general engine model due to degradation and individual characteristic difference ([1]). A data-guided modelling for individual engine in-service is considered important. Although a nonlinear model gains a lot of interests from academic research, its computation complexity is too high to be used on-board. A linear model is still very valid but should be accurate enough. Auto Regressive eXogenous (ARX) model is good for static modelling and can be easily identified by the Least Square Estimation (LSE) [2]. However, as an Equation Error (EE) model, ARX is biased for dynamic model ([3], [4], [5]) and would fail to detect incipient faults. Therefore, Output Error (OE) model is widely used in GTE modelling ([1]). The bottle neck for OE model is that its identification is a highly nonlinear task, which challenges the on-board application.

On the fault detection, the Robust Fault Detection Observer (RFDO) has received much attention during the last two decades [6], [7]. Eigenvalue assignment has been successfully applied on observer design ([8], [9]). From the viewpoint of computation complexity, the evaluation of robustnes/sensitivity criteria is the most time-consuming task in the RFDO design, because of the feature of iterative optimisation. A lot of performance indices are proposed, such as H_2, H_∞ , H_2/H_∞ ([6]), H_- /H_∞ ([10]). However, the computation of H_2, H_∞ and H_- requires an integral or grading over the whole frequency, which is too complex to be acceptable for the on-board application. In practice, a satisfactory solution can only be found by a tradeoff between the performance and computation costs.

In this paper, we propose a new performance index under the assumption that the disturbance is band-limited. The disturbance frequency is estimated from the Fast Fourier Transformation (FFT)-based residual spectrum analysis and then such information is integrated into the index evaluation and optimisation. As a mature algorithm, FFT needs far less computation than that of H_∞-based methods. During the optimisation stage, an left-eigenvector based pole assignment method is used. As illustrated in the application to a GTE fault detection, a significant improvement in sensor fault detection has been achieved. As far as we know, combining the frequency estimation and eigenvalue optimisation was seldom established.

II. PROBLEM FORMULATION

In modern aero engines, as shown in Fig. 1, control systems are usually organized as dual-lane systems with two sets of parallel sensors and controllers ([11]). Consider the condition monitoring of a two shafts GTE, the relationship between the fuel flow W_f and the shaft speeds (low pressure shaft speed N_{lp}, high pressure shaft speedN_{hp}) provides important information on health states of engines.

As an information redundancy exists within such two sets of duplicated hardware, some fault detection is possible. However, it is impossible either to decide which lane is in failure, or to detect an actuator or component fault. In order to detect these faults, a mathematical model acting as the third 'virtual' lane is introduced. It is worthy noting that,

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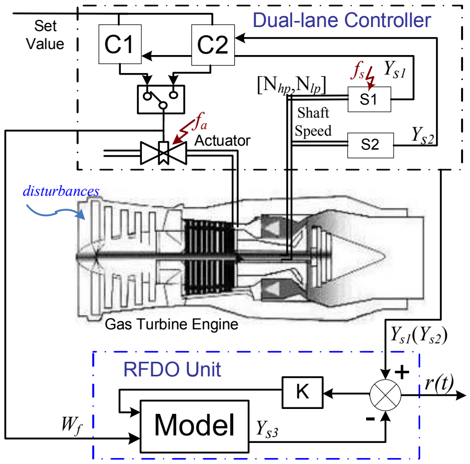

Fig. 1. Dual-lane in modern gas turbine engines.

the model in Fig. 1 has a 'parallel' structure (output error configuration).

If the system is corrupted by the disturbances and faults, the system model becomes

$$\begin{cases} x(t+1) = Ax(t) + B[u(t) + f_a(t)] + B_d d(t) \\ y(t) = Cx(t) + f_s(t) \end{cases} \tag{1}$$

where where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^r$ the output (shaft speeds) and $u \in \mathbb{R}^p$ the plant input (fuel flow rate). Usually, $f_a(t) \in \mathbb{R}^p$, $f_s(t) \in \mathbb{R}^r$ are actuator and sensor fault vectors, respectively. $B_d \in \mathbb{R}^{n \times n_d}$ is known as disturbances distribution matrix. The disturbance $d(t) \in \mathbb{R}^d$ can be caused by the variation of environment (ambient pressure, air humidity, etc.) or flight conditions. $d(t)$ can also representant the the model uncertainty. For example

$$d(t) = \Delta A x(t) + \Delta B u(t) \tag{2}$$

where $\Delta A, \Delta B$ are the parameter uncertainties. In this paper, we assume the disturbance caused by model uncertainties is relatively small and the main disturbance is band-limited.

For system (1), the robust fault detection observer under consideration can be constructed by

$$\begin{cases} \hat{x}(t+1) = A\hat{x}(t) + Bu(t) + Kr(t) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{3}$$

where $r(t) = y(t) - \hat{y}(t)$ is the residual, K is the gain to be determined.

Define the state estimation errors $e(t) = x(t) - \hat{x}(t)$, the estimation error and residual dynamics takes the form

$$\begin{cases} e(t+1) = (A-KC)e(t) + Bf_a(t) - Kf_s(t) + B_d d(t) \\ r(t) = Ce(t) + f_s(t) \end{cases} \tag{4}$$

By taking z-transformation of (4), the observer behaviour can be described by the following transfer function matrices (TFMs)

$$r(z) = G_f(z) f(z) + G_d(z) d(z) \tag{5}$$

where

$$G_f(z) = [G_a(z)\ G_s(z)], \quad f(z) = [f_a^T(z)\ f_s^T(z)]^T \tag{6}$$

and

$$\begin{cases} G_a(z) &=& C(zI - A + KC)^{-1}B \\ G_s(z) &=& -C(zI - A + KC)^{-1}K + I_{r\times r} \\ G_d(z) &=& C(zI - A + KC)^{-1}B_d \end{cases} \tag{7}$$

It can be seen from (5) that, due to the existences of disturbances, $r(k)$ is not zero even if no faults occur. The effects of disturbances act as a source of false alarms. Therefore, such negative effects need to be attenuated as much as possible, and the effects of faults should be enlarged. The solution to on-board condition monitoring and fault detection can be stated as a 3-step procedure as follows:

- **Dynamic Modelling** Collect the engine data W_f and $[N_{lp}, N_{hp}]$, and check identifiability, remove offset. Then a linear dynamic model is identified.
- **RFDO design** Determine the optimal K such that $||G_d(z)||$ is minimised and $||G_f(z)||$ is maximised.
- **Conditon Monitoring** After the modelling and observer design finished (it can take 10-20 sec), then for some time (up to several hours) this scheme is used to detect whether any fault or abnormal condition occur.

III. OE Modelling

The challenge for on-board modelling of GTEs is the development of a fast iterative OE model identification algorithm. In this section, the standard *Nonlinear Least Square* (NLS) algorithm is used. The proposed algorithm is formulated by first obtaining a recursive expression for the gradient calculation of an OE model. Then, the Hessian matrix is approximated by the first derivatives to further accelerate the identification speed.

By linearising the detailed thermodynamic GTE model, the dynamic model of $W_f \to [N_{lp}, N_{hp}]$ can be mathematically described by two n-th order Single-Input and Single-Output (SISO) submodels:

$$\hat{y}_i(t) = \frac{\sum_{j=1}^{n} \beta_{i,j} q^{-j}}{1 + \sum_{j=1}^{n} \alpha_{i,j} q^{-j}} u(t) = \theta_i^T \cdot \hat{z}_i(t) \tag{8}$$

where $i = \{1, 2\}$ and $\theta_i = [\alpha_{ij} \ldots \beta_{ij}]^T, j = 1 \ldots n$ is the parameter vector ($\theta_i \in \mathbb{R}^{2n}$) and $\hat{z}_i(t) = [\hat{y}_i(t-1) \ldots \hat{y}_i(t-n)\ u(t-1) \ldots u(t-n)]^T$, ($\hat{z}_i(t) \in \mathbb{R}^{2n}$). Note that $\hat{z}_i(t)$ in (8) contains the model prediction $\hat{y}_i(t-j)$ etc., in stead of actual plant output $y_i(t-j)$. This is the so-called OE model and is more difficult to be identified than the common EE model. Because the model prediction $\hat{y}_i(t)$ not only depends on the parameters, but also on the previous prediction $\hat{y}_i(\tau), \tau < t$, the error surface becomes a highly nonlinear function ([3]). This is the main difference from EE model.

In OE identification of the i-th submodel, the objective is to minimise the following objective function:

$$\min_{\theta_i \in \Re^{2n}} E(\theta_i) = \sum_{t=1}^{N} (y_i(t) - \theta_i^T \cdot \hat{z}_i(t))^2 \tag{9}$$

where N is the total number of data pair. In NLS, the update of parameters is given by

$$\theta_i(k+1) = \theta_i(k) - \eta^* \cdot [\mathbf{R}_i(k)]^{-1} \cdot \frac{\partial E(\theta_i(k))}{\partial \theta_i} \tag{10}$$

where η^* is a series of positive decreasing scalars, in term of step size, and $\mathbf{R}(k)$ is a $2n$-by-$2n$ positive definite matrix to modify the search direction. Here, $\frac{\partial E(\theta_i(k))}{\partial \theta_i} \in \mathbb{R}^{2n}$ denotes the derivatives of $E(\theta_i)$ with respect to θ_i at $\theta_i = \theta_i(k)$.

A. Calculation of $\partial E(\theta_i(k))/\partial\theta_i$

The gradient calculation is the basis of NLS algorithm. The correctness of the gradient calculation affects the search performance significantly. In our identification, the certain characteristics of model prediction dependence are exploited to form the iterative calculation of gradient.

For equation (8), note that $\hat{z}_i(t)$ involves previous model prediction $\hat{y}_i(\tau), \tau < t$ and then is also a function of parameters θ_i. Therefore, $\frac{\partial \hat{z}_i(t)}{\partial \theta_i}$ is not zero. Consider this and let $g_i(t) = \frac{\partial \varepsilon_i(t)}{\partial \theta_i}$ denote the local gradient information at sample time t in the k-th iteration, it follows that

$$
\begin{aligned}
g_i(t) &= \frac{\partial(y_i(t) - \hat{y}_i(t))}{\partial \theta_i} \\
&= -\frac{\partial(\theta_i^T(k)\cdot \hat{z}_i(t))}{\partial \theta_i} \\
&= -(\hat{z}_i(t) + \frac{\partial \hat{z}_i(t)}{\partial \theta_i}\cdot \theta_i(k)) \\
&= -\hat{z}_i(t) - \big[\frac{\partial \hat{y}_i(t-1)}{\partial \theta_i}\cdots\frac{\partial \hat{y}_i(t-n)}{\partial \theta_i}\cdots \\
&\qquad \frac{\partial u(t-1)}{\partial \theta_i}\cdots\frac{\partial u(t-m)}{\partial \theta_i}\big]\cdot \theta_i(k) \\
&= -\hat{z}_i(t) + [g_i(t-1)\cdots g_i(t-n)\ 0\cdots 0]\cdot \theta_i(k)
\end{aligned} \tag{11}
$$

As such, it can be obtained that

$$\frac{\partial E(\theta_i(k))}{\partial \theta_i} = 2\sum_{t=1}^{N} \varepsilon_i(t)\cdot g_i(t) \tag{12}$$

Remark 1: Compared to commonly used gradient calculation that usually contains only the first term on the right-hand side of Eq. (11), the iterative computing of gradient $g_i(t)$ is the key idea to improve the identification speed. Furthermore, the estimation bias in EE model is removed by the definition of OE error, and the dependency of errors is solved by computing the local gradient $g_i(t)$ in a iterative form, (11).

B. Approximation of $\mathbf{R}_i(k)$

At k-th step of θ_i updating, since $g_i(t), (1 < t < N)$ have been calculated for every time instant t, the Jacobian matrix of the objective function (9) can be straightly obtained by transforming $g_i(t)$ into matrix form:

$$\mathbf{J}_i(k) = [g_i^T(1)\ g_i^T(2)\cdots g_i^T(N)]^T \tag{13}$$

where $\mathbf{J}_i(k) \in \mathbb{R}^{N\times 2n}$. With the aid of the second-order Taylor expansion of objective function (9), it is well known that $\mathbf{R}_i(k)$ in (10) can be approximated by the Hessian matrix which consists of the second derivatives of (9). Computing the second derivatives is time consuming and not suitable for on-board processors. In order to avoid the computation of the second derivatives, the Hessian can be further approximated by the Jacobian (the 1st derivatives).

$$\mathbf{R}_i(k) \approx \mathbf{H}_i(\theta_i(k)) \approx \mathbf{J}_i^T(k)\cdot \mathbf{J}_i(k) + \varepsilon(t)\frac{\partial^2 \varepsilon_i(t)}{\partial \theta_i \partial \theta_i^T} \tag{14}$$

If the second term on the right-hand side of (14) is ignored, it becomes the so-called Gauss-Newton method.

Remark 2: The main advantage in this approximation is that only the first-order derivatives need to be calculated. Since such derivatives have been available in the computation of local gradients, no extra computation are required.

Remark 3: The search direction is further modified from inverse gradient by a approximated Hessian matrix. Therefore, the search direction and convergence speed should be better than those deepest descent methods.

IV. Frequency-dependent Performance Indices

After each SISO model (8) is identified, they are transformed into state-space representations which are useed to design fault detection observers. The design of a robust fault detection observer is a 2-step task: (1) the definition of the performance index; (2) the optimal selection of K.

A. Robustness Performance Index

Many robustness performance indices have been proposed during the last two decades. H_∞-norm performance has been widely accepted. However, its computation cost is too high to be affordable for on-board processors. This is because the H_∞-norm requires griding over the whole frequency $[0, \pi]$, computing the singular value and finding the largest one. Based on the observation that the main disturbances in GTE systems are frequency band limited, in order to reduce the computation of the observer design, a modified frequency dependent robustness index is proposed here.

$$\min \|G_d(z)\|_{z=e^{jw_d}} \tag{15}$$

where the disturbance is assumed mainly concentrated at some frequency w_d, $0 \le |w_d| \le \pi$. Hence, the time-consuming computation of H_∞-norm over $[0, \pi]$ is avoid.

Similar to H_∞ theory, here the disturbance is still unknown which raises a new problem: how to estimate w_d. It is well known that a SISO system does not change the output frequency from input frequency. It is also true for discrete-time observer and can be stated as :

For a discrete system (1) corrupted by a band-limited disturbance $d(t)$ (whose main frequency component is at w_d), if observer (4) is stable, then at steady state, by collecting residual $r(t)$ for long time enough, the main frequency component w_f in residual spectrum is equivalent to w_d.

$$\{w_r | w_r \in spec(r(t))\} \subseteq \{w_d | w_d \in spec(d(t))\}$$

where, the (power) spectrum of $r(k)$ is defined as the sum of its DFT (Discrete Fourier Transform) sequence $|\mathbf{R}_i[n]|$ squared, $\text{spec}(\mathbf{r}(t)) = \sum_{i=1}^{i=p} |R_i[n]|^2$.

It follows that the index (15) can be computed as

$$\min J_1 = \|G_d(z)\|_{z=e^{jw_r}} \tag{16}$$

B. Sensitivity Performance Index

In order to avoid the computation of $||G_f(z)||_\infty$, similarly, the frequency information of faults should be taken into account. In the frequency domain, an incipient fault comprises mainly low frequency components. For abrupt faults, high frequency contents only exist at the time instant when faults begin, and it is almost constant thereafter. Therefore, the steady state gain $G_f(z)|_{z=1}$ is the most important factor in fault detection. [7] proposed a strong fault detectability condition: $\|G_f(s)\|_{s=0} \neq 0$. In this paper, it is proposed that the $\|G_f(z)\|_{z=1}$ index should be maximised for increasing the fault significance, which gives

$$\max J_2 = \|G_f(z)\|_{z=1} \tag{17}$$

Combining the robustness index (16) and sensitivity index (17) leads to the whole performance index as:

$$\min J = \frac{J_1}{J_2} = \frac{\|G_d(z)\|_{z=e^{w_r}}}{\rho + \|G_f(z)\|_{z=1}} \tag{18}$$

where ρ is a small positive real number that guarantees the denominator will not be zero.

Remark 4: Since the variable z in (18) is assigned specific values w_r and 1, respectively, the norm computation of a TMF is replace by the norm calculation of a constant matrix. Compared to the H_∞ TMF-norm that requires calculation over the whole frequency $[0, \pi]$, and the computation of a real coefficient matrix is very low.

Remark 5: As the frequency information is incorporated into the new index (18), the resulting observer is optimal for attenuating such a disturbance. In most applications, such an observer has a better performance than the conservative H_∞ observer.

V. Fast RFDO Design

As shown in a lot of literature ([7], [12]), eigenvalue assignment method is able to assign the eigenvalue arbitrarily to the desired places and satisfy certain additional performance index. It is very suitable for solving the RFDO problem. The idea of the eigenstructure assignment is to assign the eigenvalues of $A-KC$ (the poles of the observer (4)) arbitrarily to the desired places $\{\lambda_i\}$ by selecting an appropriate $K \in \mathbb{R}^{n\times p}$. With the aid of the modified performance index (18), a left eigenvectors assignment observer design method is proposed.

Under some weak conditions [12], the gain matrix K of observer (3) and the TFMs $G_d(z)$, $G_f(z)$ can be parametrized by eigenvalues λ_i and free parameters q_i, as shown in the following **Lemma**.

Lemma Let A, C be observable, then, for any group of scalars λ_i, $i = 1, 2, \ldots, n$, the gain matrix K can always be parameterised as:

$$K = L^{-1}Q \tag{19}$$

where $Q = [q_1\ q_2\ \ldots\ q_n]^T$ is free parameters and $L \in \mathbb{R}^{n\times n}$ is composed of the left eigenrows l_i of $A-KC$, corresponding to the eigenvalue λ_i respectively. L is determined by

$$L = \begin{bmatrix} l_1^T \\ \vdots \\ l_n^T \end{bmatrix} = \begin{bmatrix} q_1^T\mathbf{C}(\mathbf{A}-\lambda_1\mathbf{I})^{-1} \\ \vdots \\ q_n^T\mathbf{C}(\mathbf{A}-\lambda_n\mathbf{I})^{-1} \end{bmatrix} \tag{20}$$

The discrete TFMs (7) can be expanded as

$$\begin{aligned} G_d(z) &= C\ R\Psi(z)L\ B_d \\ G_f(z) &= [CR\Psi(z)LB \quad -CR\Psi(z)LBL^{-1}Q + I] \end{aligned} \tag{21}$$

where

$$\Psi(z) = \text{diag}(\frac{1}{z-\lambda_1}, \ldots, \frac{1}{z-\lambda_n}) \tag{22}$$

and

$$R = L^{-1} = (r_1, r_2, \ldots, r_n) \tag{23}$$

Proof The proof for parametric expression of gain matrix K in continuous domain is standard, see [12]. For the parametric expression of TFMs, it is similar to the Lemma 10.2 in [12]. By replacing the s-transformation with the z-transformation, the inverse of any discrete TFM $(zI - A + KC)^{-1}$ can be expanded as

$$(zI - A + KC)^{-1} \quad = \tfrac{r_1 l_1^T}{z-\lambda_1} + \ldots + \tfrac{r_n l_n^T}{z-\lambda_n} \tag{24}$$

Substituting (24) into (7) gives the parametric expressiones of $G_d(z)$ and $G_f(z)$. Then rewriting it into matrix form gives (21). **Q.E.D.**

Base on the discussion above, the solution to discrete RFDO under quasi-stationary disturbances can now be states as the follows:

If the main frequency contents of residuals $r(k)$ can be estimated at w_r, then minimising the performance index

$$J = \frac{\left|\left|CR\Psi(z)LB_d\right|\right|_{z=e^{jw_r}}}{\rho + \left|\left|CR\Psi(z)LB \quad -CR\Psi(z)LBL^{-1}Q + I\right|\right|_{z=1}} \tag{25}$$

gives the optimal gain matrix $K = L^{-1}Q$ such that the effect of the disturbance $d(k)$ is attenuated and the sensitiveness of the residual to faults is enhanced at the greatest extent.

VI. Application and Results

To illustrate the proposed observer design approach, this section presents the results of application to incipient fault detection of a gas turbine engine. Real engine fuel flow data gathered from the engine test-bed are used ([11]). 1500 data pairs were collected at sampling rate 40Hz from normal engine operation.

A. System Identification

For simplification, only the identification of the $W_f \rightarrow N_{lp}$ model is shown here. The same process can be applied to $W_f \rightarrow N_{hp}$. In order to show the iterative search clearly (see Fig. 2), a first order model is used for illustration. It can be seen from the contour of OE objective function surface in Fig. 2 that the objective function of OE model is nonlinear with a narrow valley around the global minimum (Δ). The initial values are $\theta_0 = [0.5\ 0.05]^T$. In the deepest gradient method, even after 200 iterations there is still a considerable distance from the minimum. For the Gauss-Newton method, the search route is obviously better whose direction points a more straightforward way. Furthermore, only 18 iterations are needed to arrive at the minimum. Compared to the deepest gradient method, the convergence speed of Gauss-Newton increases dramatically and the parameters' estimates are more accurate.

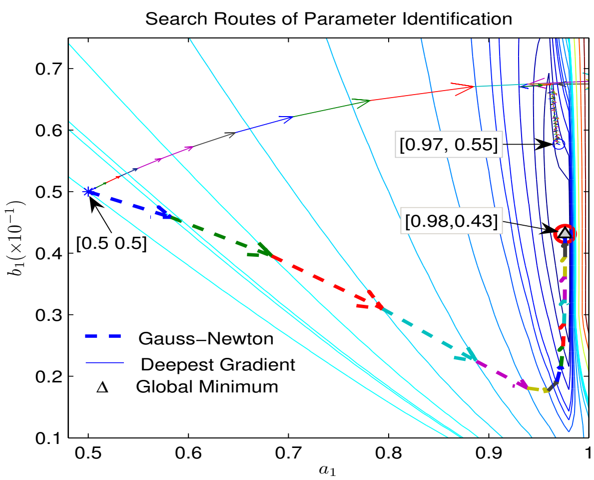

Fig. 2. Searching routes of Deepest Gradient method (solid) and Gauss-Newton method (dashed).

In the real application, the second order model is identified, as listed in Table I. It can be seen that only 103 iterations were involved in the modified Gauss-Newton method.

TABLE I

LONG-TERM PREDICTION FOR 2ND ORDER MODEL

Methods	Iter-ations	on Training Data STDs	on Training Data MSEs	on Validation Data STDs	on Validation Data MSEs
LSE	1	3.41291	1.16613	2.87529	10.3296
ARX	1	3.93632	1.55375	3.05683	11.5071
RIV	N/A	1.25340	1.81345	1.97333	5.19088
ExhSrch [a]	10^6	1.25624	1.78773	1.95562	5.16392
OE1 [b]	20000	2.30235	5.39615	2.74971	9.22707
OE2 [c]	103	1.25624	1.78773	1.95563	5.16391

[a]exhaustive search method
[b]deepest gradient search method
[c]Gauss-Newton search method

Then, a reduced order model of the GTE is formed by converting the two SISO submodels into the state space with parameters:

$$A = \begin{bmatrix} 0.9769 & 0.0038 \\ 0.0936 & 0.9225 \end{bmatrix}, B = \begin{bmatrix} 2.1521 \\ 3.8186 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{26}$$

The input and outputs signals and model predictions are plotted in Fig.3.

B. Fault Detection

In this application, the detection of sensor faults is the focus. and both abrupt and incipient faults are considered.

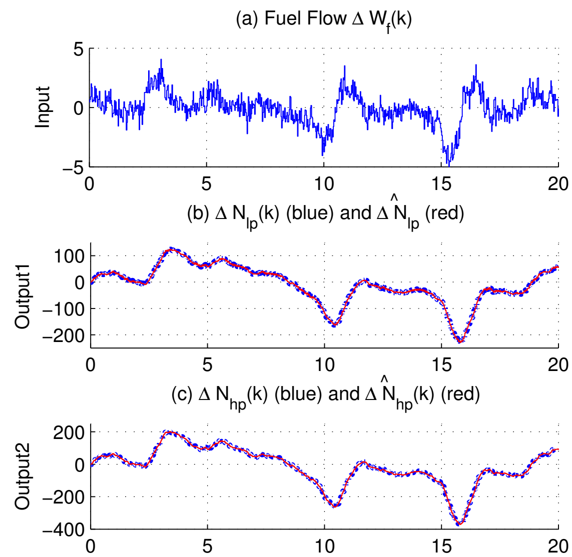

Fig. 3. The input, the measured outputs and their model predictions. The data has been subtracted from the operating point equilibrium at steady state.

The disturbance model is assumed as

$$B_d = [0.1510 \quad 0.0500]^T \tag{27}$$

The disturbances injected to the systems are

$$\mathbf{d}(k) := \begin{cases} d_1(k) = \sin(2k) + \cos(2.1k + \frac{\pi}{4}) \\ d_2(k) = \sin(2.1k) \end{cases} \tag{28}$$

and its 512-point FFT based spectrum is shown in Fig. 4 (a). In order to estimate the disturbance frequency, a gain matrix K_0 is first constructed via *place(A',C', [-0.5 0.5])'* and its residual spectrum is shown in Fig. 4 (b). The coincidence between plot (a) and (b) verifies that the disturbance frequency can be estimated by residual spectrum.

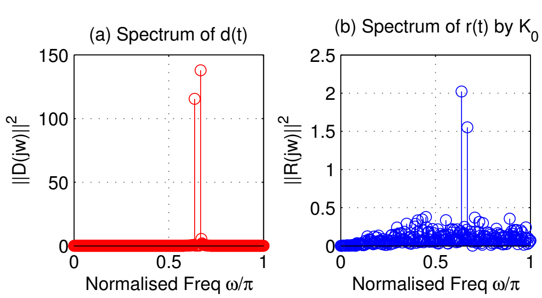

Fig. 4. (a) Disturbance Spectrum and (b) its frequency estimation through 512-point FFT-based residuals spectrum analysis

During the eigenstructure assignment, the desired poles region are set as $\text{norm}(\lambda_i) < 0.75$. The initial values of λ_i are $[-0.5 \ 0.5]$ which ensure that they are distinct from the GTE poles [0.9828, 0.9166]. The optimal gain matrix K given by the eigenstructure assignment algorithm is

$$K_{opt} = \begin{pmatrix} 3.3769 & -11.0584 \\ 0.9516 & -2.8394 \end{pmatrix} \tag{29}$$

with optimal poles [0.6519, 0.7100] and performance index $J = 0.0005583$. For comparison, K_{place} are computed by using *place* command provided by MATLAB Control Toolbox,which gives

$$K_{place} = \begin{pmatrix} 0.3250 & 0.0038 \\ 0.0936 & 0.2125 \end{pmatrix} \quad (30)$$

where K_{place} gives the identical eigenvalues as K_{opt}.

C. Detection of an Abrupt Fault

A small abrupt sensor fault in the speed sensor for low pressure shaft is simulated by adding a small step (0.25) to the output signal N_{lp}. The simulated fault takes place at 10.05s when the outputs reach the maximum negative values. Fig. 5 shows the norm of the residual vector. The observer K_{place} fails to detect such an abrupt fault, as there are no apparent changes of $\|r(k)\|_{place}$ after fault happening. However, K_{opt} gives a significant step increase in its residual immediately after the fault occurring.

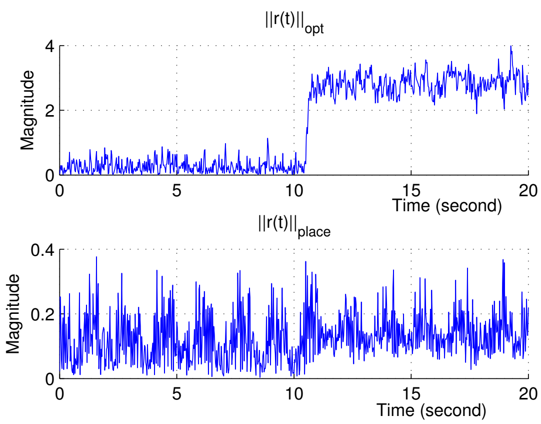

Fig. 5. Residuals of K_{opt} and K_{place} in the case of abrupt sensor fault with amplitude 0.25

D. Detection of an Incipient Fault

The ability of the proposed RFDO to detect drifting faults is also demonstrated here. Such faults are extremely difficult to detect immediately from a simple visual inspection of the output signals. To simulate the incipient fault of sensor gain drift of 0.5 units per second, the fault function added to the N_{lp} is given by $f(t) = 0.5(t - 10.05), (t \geqslant 10.05)$. The residuals are shown in Fig. 6.

Similar to the abrupt fault case, K_{opt} shows a great improvement on detecting incipient faults. From the view point of fault detection delay, K_{opt} can give fault alarm less than 3 seconds after the fault happening and no false alarm thereafter. However, even 10 seconds later, the K_{place} fails to detect the fault. This verifies that K_{opt} can detect an incipient fault earlier and clearer.

VII. Conclusion

This paper proposes a dynamic modelling and discrete robust observer design approach for fault detection of GTEs, in which the disturbances can be assumed band-limited. A

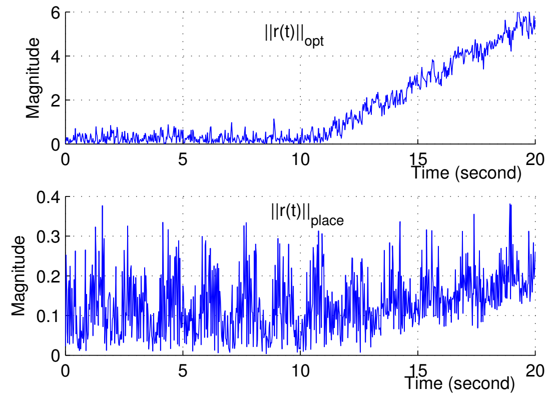

Fig. 6. Residuals of K_{opt} and K_{place} in the case of incipient sensor faults

recursive expression of the gradient calculation is presented and the Hessian matrix is approximated by the first derivatives to further accelerate the identification speed. For a better disturbance attenuation, the disturbance frequency is estimated through the FFT-based residual spectrum. Then a simplified but effective robustness/sensitivity index is proposed and the lefteigenvector-based eigenvalue assignment is used to find the optimal gain K. The advantages of the proposed method are that less computation is required and the fault detection performance is remained.

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